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ABSTRACT

Sauer recently proposed a new approach to hyperbolic problems involving more than two independent variables. In the case of plane, unsteady problems depending on time t and Cartesian coordinates x,y he finds the characteristic lines of the governing equations in the x,t plane and develops a finite difference scheme based on these lines and lines parallel to the y axis. In this way a two dimensional unsteady problem is solved as a sequence of one dimensional problems in planes y = constant. This scheme is much simpler than earlier schemes based on bicharacteristics. The present report describes the scheme and discusses its application to underwater shock wave problems.

1. INTRODUCTION

Many important problems connected with the theory of underwater explosions depend on two space variables and time. In their non-linear formulation these can be solved numerically by the method of characteristics. When more than two independent variables arise, the method of characteristics can be presented in a variety of forms and it appears now that some of the early numerical schemes proposed for the method were unnecessarily complicated. The present report describes some schemes which have recently been developed in Germany, with particular emphasis on Sauer's Near Characteristics Method (ref. 1), which seems to be well suited for application to underwater problems. After considering the method in its general aspects, it is worked out in detail for application to two problems: (1) the propagation of a spherical explosion in an ocean with a variable density gradient; (2) the reflection and refraction of a spherical explosive wave at the ocean surface.

Methods of Characteristics developed earlier for two dimensional unsteady flow problem were based wholly on the use of bicharacteristics. Those proposed by Thornhill (ref. 2) and Butler (ref. 3) were based on the following approach. Starting with the Eulerian equations for unsteady flow the equations defining bicharacteristics are determined, together with the compatibility conditions satisfied along them. It is then assumed that initial data are prescribed on some space-like initial surface. To construct conditions at a new point, bicharacteristic lines are drawn backwards through the point to intersect the initial surface. Along each of these bicharacteristic lines the compatibility conditions are written as difference relations. These are simultaneous equations to determine the values of the unknowns at the new point. As many bicharacteristic relations are used as these are unknown dependent variables. Butler has applied his method to calculate the hypersonic flow past a pointed unsymmetrical body (using the analogy between hypersonic small disturbance theory and unsteady flow theory). The calculation was a notable achievement, although the method is more complicated than those developed later. The chief disadvantage of the method is that the directions along which difference relations are established follow no particular pattern. Further, it is questionable whether bicharacteristic relations on Mach cones can be used more than twice between an initial surface and a new point.

Methods were developed for steady flow problems by Coburn and Dolph (ref. 4) and Holt (ref. 5) which only employ two bicharacteristic relations at each point. These, together with a third non-characteristic relation and the streamline direction, are related in a definite way to prescribed conditions on an initial surface.

The methods developed recently in Germany also use only two bicharacteristic directions (or in the case of Sauer's method two general sections of the local Mach conoid) but the choice of these is related to the local flow geometry rather than to initial conditions. The first of these methods was proposed by Bruhn and Haack (ref. 6) and later modified by Schaetz (ref. 7). The basic approach is to replace the derivatives occurring in the equations of motion with respect to the original independent variables (for example two Cartesian coordinates and time) by directional derivatives. These are chosen so that, in the transformed equations, the number of directions is reduced to a minimum. When these transformed equations are replaced by difference equations, the latter can be solved with the minimum of algebraic operations and hence with the least expense of programming time.

In the method developed by Schaetz for unsteady flow the equations of motion are transformed to coordinates based on the following local directions:

(1) the direction of the resultant velocity.

(2) the direction of the particle path.

(3) bicharacteristic directions perpendicular to the resultant velocity direction.

Schaetz calls his method optimal since it employs the minimum number of directions. It is a much simpler scheme than those based entirely on bicharacteristics - of the four directions used three are in the same plane. However, these directions do change with the flow direction. Sauer's Near Characteristics Method (ref. 1) avoids this drawback. It is also an optimal method (using only four directions) but uses a fixed coordinate direction as one of these. The plane containing the other three directions is always in a fixed direction (normal to the fixed coordinate direction), two directions are sections of the local Mach cone on this plane, the third is the projection of the velocity vector on the plane.

Using Sauer's approach the method of characteristics for two dimensional unsteady problems reduces to applying the one-dimensional method in a sequence of coordinate planes. This makes it easier to control than methods based on many spatial directions and easier to use for interpolation.

The advantages of Sauer's method are illustrated by a discussion of its application to the two problems mentioned above. The first is most conveniently referred to spherical polars and is partly a finite difference method while the second is an initial motion problem solved by expansion in series and referred to cylindrical polar coordinates.

2. GENERAL FORM OF SAUER'S NEAR CHARACTERISTICS METHOD

Consider a system of m quasilinear partial differential equations of the first order

$$A_{kj} \frac{\partial uj}{\partial t} + b_{ij}^{k} \frac{\partial uj}{\partial xi} = h^{k} \qquad 2.1$$

where

$$j = 1, \ldots. m, \qquad k = 1, \ldots. m, \qquad i = 1, \ldots. n$$

and n + 1 is the number of independent variables (including the time t). The coefficients a_{kj}, b_{ij}^{k} and h^{k} are functions of t, x and n.

We rewrite equations 2.1 so that only derivatives with respect to t and x_1 occur on the left while all remaining derivatives, together with terms h^k are put on the right. We then multiply each equation by a factor σ_k and add the resulting equations. We shall obtain the equation

$$A_1 \frac{\partial u_1}{\partial t} + B_1 \frac{\partial u_1}{\partial x_1} + A_2 \frac{\partial u_2}{\partial t} + B_2 \frac{\partial u_2}{\partial x_1} + \ldots + A_m \frac{\partial u_m}{\partial t} + B_m \frac{\partial u_m}{\partial x_1} = H \qquad 2.2$$

where

$$A_j = \sigma_k a_{kj}$$

$$B_j = \sigma_k b_{ij}^{k}$$

H is a function of $\sigma_1, \ldots \sigma_m$, t , $x_1 \ldots . x_n$ and contains derivatives with respect to $x_2, \ldots . x_n$.

We now choose $\sigma_1, \ldots \sigma_m$ so that the directional derivatives occurring on the left of equation 2.2 are all the same. The common direction is defined by

$$\frac{dx_1}{dt} = \frac{B_1}{A_1} = \ldots\ldots\ldots\ldots\ldots\ldots = \frac{B_m}{A_m} = \tau$$

$$dx_2 = \ldots\ldots\ldots\ldots\ldots\ldots = dx_n = 0 \qquad 2.3$$

Equation 2.3 then leads to m equations

$$B_1 - \tau A_1 = B_2 - \tau A_2 = \ldots\ldots\ldots = B_m - \tau A_m = 0 \qquad 2.4$$

to determine $\sigma_1, \ldots. \sigma_m$. These will give non-trivial solutions only if the determinant formed by the coefficients of $\sigma_1, \sigma_2 \ldots.. \sigma_m$ is zero. The determinantal condition gives r values of $\tau_\rho (\rho = 1, \ldots r)$. In the case of hyperbolic equations $2 \leq r \leq m$ and the introduction of the values τ_ρ into equations 2.4 leads to m independent equations of type 2.2. These may now be written

$$A_1^k \left(\frac{du_1}{dt}\right)_\rho + \ldots\ldots\ldots + A_m^k \left(\frac{du_m}{dt}\right)_\rho = H_\rho \qquad 2.4$$

$$\rho = 1, \ldots r. \quad k = 1, \ldots m$$

Equations 2.4 are in every way equivalent to the original equations 2.1 and are transformations of these in terms of near characteristics. The effect of the use of near characteristics is to reduce the number of directional derivatives in each of the original equations by one, a significant simplification. In the case of one dimensional unsteady motion, of course, this transformation results in each equation containing one directional derivative only.

3. UNDERWATER EXPLOSIONS IN AN OCEAN WITH A VERTICAL DENSITY GRADIENT

This problem is conveniently referred to spherical polar coordinates. The Eulerian equations of motion, for rotational flow, may be written

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{w^2}{r} - \frac{w}{r}\frac{\partial u}{\partial \theta} \qquad 3.1$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} = -\frac{w}{r}\frac{\partial w}{\partial \theta} - \frac{uw}{r} - \frac{1}{\rho r}\frac{\partial p}{\partial \theta} \qquad 3.2$$

$$a\frac{\partial u}{\partial r} + \frac{1}{\rho a}\frac{\partial p}{\partial t} + \frac{u}{\rho a}\frac{\partial p}{\partial r} = -\frac{a}{r}\frac{\partial w}{\partial \theta} - \frac{2ua}{r} - \frac{wa}{r}\cot\theta - \frac{w}{r\rho a}\frac{\partial p}{\partial \theta} \qquad 3.3$$

Here, t is the time measured from the instant of initiation, r is distance measured from the center of the explosion and θ is the angle between a vertical axis through the center and the radius.

Multiply 3.1 by σ_1, 3.2 by σ_2, 3.3 by σ_3 and add

Then

$$\sigma_1\frac{\partial u}{\partial t} + (\sigma_1 u + \sigma_3 u)\frac{\partial u}{\partial r} + \left\{\frac{\sigma_3}{\rho a}\frac{\partial p}{\partial t} + \left(\frac{\sigma_3}{\rho a} + \frac{\sigma_1}{\rho}\right)\frac{\partial p}{\partial r} + \sigma_2\frac{\partial w}{\partial t} + \sigma_2 u\frac{\partial w}{\partial r} = H \qquad 3.4$$

where H is the sum of the terms on the right of the factored equations. The condition that the directional derivatives on the left of 3.4 should all be the same is

$$\frac{\sigma_1 u + \sigma_3 u}{\sigma_3} = \frac{\sigma_1 a + \sigma_3 u}{\sigma_3} = \frac{\sigma_2 u}{\sigma_2} = \tau$$

Hence,

$$\begin{aligned} (u - \tau)\sigma_1 \quad & + a\sigma_3 = 0 \\ a\,\sigma_1 \quad & + (u - \tau)\sigma_3 = 0 \\ (u - \tau)\sigma_2 \quad & = 0 \end{aligned}$$

For non-trivial values of σ_1, σ_2, σ_3, τ must satisfy the equation

$$(u - \tau)\left\{(u - \tau)^2 - a^2\right\} = 0$$

Hence

$$\tau = u \qquad 3.5$$

or

$$\tau = u \pm a \qquad 3.6$$

The equations of the near characteristics are then

$$\frac{dr}{dt} = u \pm a \quad \text{(Traces of Mach Lines)} \qquad 3.7$$

$$\frac{dr}{dt} = u \quad \text{(Projections of Particle Paths)} \tag{3.8}$$

Denote directional derivatives (w.r.t. time) in the three characteristic conditions by suffices 1, 2, and 3 respectively. Thus,

$$\left(\frac{d}{dt}\right)_1 = \frac{\partial}{\partial t} + (u + a)\frac{\partial}{\partial r}$$

$$\left(\frac{d}{dt}\right)_2 = \frac{\partial}{\partial t} + (u - a)\frac{\partial}{\partial r}$$

$$\left(\frac{d}{dt}\right)_3 = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$$

Then the near characteristic forms of equations 3.1, 3.2, 3.3 are obtained by putting $\sigma_1 = 1$, $\sigma_3 = \pm 1$, $\sigma_2 = 0$; $\sigma_1 = \sigma_3 = 0$, $\sigma_2 = 1$. We obtain

$$\left(\frac{du}{dt}\right)_1 + \frac{1}{\rho a}\left(\frac{dp}{dt}\right)_1 = \frac{w^2}{r} - \frac{w}{r}\frac{\partial u}{\partial \theta} - \frac{a}{r}\frac{\partial w}{\partial \theta}$$
$$- \frac{2ua}{r} - \frac{wa}{r}\cot\theta - \frac{w}{r\rho a}\frac{\partial p}{\partial \theta} \tag{3.9}$$

$$\left(\frac{du}{dt}\right)_2 - \frac{1}{\rho a}\left(\frac{dp}{dt}\right)_2 = \frac{w^2}{r} - \frac{w}{r}\frac{\partial u}{\partial \theta} + \frac{a}{r}\frac{\partial w}{\partial \theta}$$
$$+ \frac{2ua}{r} + \frac{wa}{r}\cot\theta + \frac{w}{r\rho a}\frac{\partial p}{\partial \theta} \tag{3.10}$$

$$\left(\frac{dw}{dt}\right)_3 = -\frac{w}{r}\frac{\partial w}{\partial \theta} - \frac{uw}{r} - \frac{1}{\rho r}\frac{\partial p}{\partial \theta} \qquad 3.11$$

To start the method we need to know the complete disturbance field at some time $t = t_o$. The determination of these initial conditions will depend on the type of explosion considered. In general we may suppose that they are given in the form

$$u = u(r, \theta) \text{ etc. at } t = t_o \qquad 3.12$$

To determine flow conditions at a later instant $t_o + \Delta t$, at a new point $Q(r_o, \theta_o)$ we proceed as follows: (1) Calculate all flow conditions at the point P, which is the projection of Q on the $t = t_o$ plane. In particular, determine the characteristic directions $\left(\frac{d}{dt}\right)_1$, $\left(\frac{d}{dt}\right)_2$, $\left(\frac{d}{dt}\right)_3$ at P. (2) At Q draw lines with these characteristic directions to intersect the plane $t = t_o$ at points A, B and C respectively. Note that these lines all lie in the plane $\theta = \theta_o$. (3) Write equations 3.9, 3.10 and 3.11 as difference relations along AQ, BQ, CQ respectively. The terms on the right of these equations are evaluated at A, B and C respectively. Solve these equations to determine the values of u, p, and w at Q.

In the general case these equations need to be supplemented by the equation of state and the condition that entropy is conserved on particle paths. The latter can be written in characteristic form

$$\left(\frac{dS}{dt}\right)_3 = -\frac{w}{r}\frac{\partial S}{\partial \theta} \qquad 3.13$$

which can be solved as a difference equation along CQ. In water the Tait equation of state

$$p = B(S) \left\{ \rho^7 - A(S) \right\} \qquad 3.14$$

may be used. In general B and A can be taken as constant and equation 3.13 can be dropped.

This crude difference process can be improved. Once first approximations to the values of flow quantities at Q have been determined equations 3.9 - 3.12 can be solved again along QA, QB, QC using mean differences.

At each stage of the integration shocks and other boundaries (e.g. contact discontinuities) must be fitted. The procedure for doing this is similar to that used in one dimensional flow but must be applied in a series of planes θ = constant.

Three sets of initial conditions can be used, corresponding to three types of explosion. The simplest corresponds to a pressurized sphere which is burst symmetrically at time $t = 0$. Secondly, we may consider a spherical charge of explosive initiated at its center; in this case the blast propagation is preceded by a spherically symmetric detonation phase. The initial field of disturbance in both these cases can be calculated by series expansions similar to those used by Holt (ref. 8) to analyse the initial behavior of spherically symmetric explosions. The coefficients of these series will now depend on θ, as well as r and t. For small times after initiation, when the vertical density gradient is small, it is convenient to use Fourier series in θ.

Thirdly, nuclear explosions should be considered. These are determined from perturbation of similarity solutions and have been treated extensively in the book by Korobeinikov, Mel'nikova and Ryazanov (ref. 9).

4. REFLECTION AND REFRACTION OF A SPHERICAL SHOCK WAVE AT THE OCEAN SURFACE

The problem of reflection of a spherical explosive wave at a solid plane surface has been solved by Vasil'ev (ref. 10). Series expansions were used in powers of time, measured from the instant the wave first hits the surface, and radial distance, measured from the initial point of contact of the wave. By retaining terms in the series up to the fourth degree the shape of the reflected shock wave and the values of pressure and density behind it can be worked out accurately up to the stage when the incident shock makes an angle of 40° with the plane surface.

This suggests the investigation of a more difficult problem, the reflection and refraction of a spherical explosive wave at a free surface, especially the surface of the ocean. If both the ocean and air above it are treated as uniform media and if a spherical explosion is detonated some distance below the surface, the resulting wave, on reaching the surface, will be partly transmitted as a shock into the atmosphere and partly reflected as an expansion wave back into the ocean. In any meridian plane this expansion wave, at a given time, is centered on the point in the surface then reached by the incident wave. The ocean surface is disturbed by the refraction process and it is necessary to determine the shape of the transmitted wave, the distortion of the ocean surface, and the field of interaction between the reflected expansion wave and the incident explosive wave (see Figure 2).

To treat this initial motion the governing equations are written in cylindrical polars.

Momentum

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} = - u\frac{\partial u}{\partial r} - \frac{1}{\rho}\frac{\partial p}{\partial r} \qquad 4.1$$

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} = -u\frac{\partial w}{\partial r} - \frac{1}{\rho}\frac{\partial p}{\partial z} \qquad 4.2$$

Continuity

$$\frac{\partial p}{\partial t} + w\frac{\partial p}{\partial z} + \rho a^2\frac{\partial w}{\partial z} = -u\frac{\partial p}{\partial r} - \rho a^2\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \qquad 4.3$$

Entropy conservation

$$\frac{\partial S}{\partial t} + w\frac{\partial S}{\partial z} = -u\frac{\partial S}{\partial r} \qquad 4.4$$

State

$$p = B(S)\left\{\rho^{\gamma} - A(S)\right\} \qquad 4.5$$

In water B and A are taken to be constants; $\gamma = 7$

In air $B = e^{(S-\bar{S})/C_v}$, $A = 0$, $\gamma = 1.4$

The z axis is taken on the vertical line through the origin of the explosion. The radial coordinate r is measured from the point on the ocean surface where the spherical shock is first in contact.

To determine the field of disturbance near the point (r, 0) it is convenient to introduce a reduced time τ. This is measured from the instant when the incident wave reaches the point (r, 0) rather than the instant of initial contact.

If U(R) is the value of the incident shock wave velocity (R is measured from the center of the explosion) the point of intersection of the incident shock with the free surface is moving with velocity

$$U_1 = \frac{dr_1}{dt} = \frac{r}{h}\left\{U(h) + \frac{1}{2}\frac{r^2}{h^2}(hU'(h) - U(h)) \ldots\right\} \qquad 4.6$$

where h is the depth of the center of the explosion

The reduced time τ is therefore,

$$\tau = t - \frac{r}{U_1} = t - t_1$$

or $$\tau = t - \frac{h}{U(h)} + \frac{1}{2}\,\frac{r^2}{hU^2(h)}\,(hU'(h) - U(h)) \ldots \qquad 4.7$$

In equations 4.1 - 4.4 change independent variables from r, z, t to r, z, τ. Then

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r} + kr\,\frac{\partial}{\partial \tau} \qquad 4.8$$

(neglecting higher order terms in r)

where

$$k = \frac{h\,U'(h) - U(h)}{h\,U^2(h)} \qquad 4.9$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} \qquad 4.10$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z} \qquad 4.11$$

The transformed equations are to be expanded in series of powers of r. To determine the lowest order terms in these expansions we may neglect terms of order r^2. Now u is of order r near r = 0 and we may therefore neglect kru (and kr $\frac{\partial u}{\partial t}$).

The equations become

$$\frac{\partial u}{\partial \tau} + w\frac{\partial u}{\partial z} + \frac{kr}{\rho}\frac{\partial p}{\partial \tau} = -u\frac{\partial u}{\partial r} - \frac{1}{\rho}\frac{\partial p}{\partial r} \qquad 4.12$$

$$\frac{\partial w}{\partial \tau} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -u\frac{\partial w}{\partial r} \qquad 4.13$$

$$\frac{\partial p}{\partial \tau} + w\frac{\partial p}{\partial z} + \rho a^2\frac{\partial w}{\partial z} = -\rho a^2\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \qquad 4.14$$

$$\frac{\partial S}{\partial \tau} + w\frac{\partial S}{\partial z} = -u\frac{\partial S}{\partial r} \qquad 4.15$$

We now write equations 4.12 - 4.14 in near characteristic form. The general linear combination of equations 4.12 - 4.14 is

$$\sigma_1\frac{\partial u}{\partial \tau} + \sigma_1 w\frac{\partial u}{\partial z} + \left(\sigma_3 + \frac{\sigma_1 kr}{\rho}\right)\frac{\partial p}{\partial \tau} + \left(\frac{\sigma_2}{\rho} + \sigma_3 w\right)\frac{\partial p}{\partial z}$$

$$+ \sigma_2\frac{\partial w}{\partial \tau} + (\sigma_2 w + \sigma_3\rho a^2)\frac{\partial w}{\partial z}$$

$$= -(\sigma_1 u + \sigma_3\rho a^2)\frac{\partial u}{\partial r} - \frac{\sigma_1}{\rho}\frac{\partial p}{\partial r} - \sigma_2 u\frac{\partial w}{\partial r} - \sigma_3\rho a^2\frac{u}{r} \qquad 4.16$$

The near characteristic conditions are determined by the conditions

$$\frac{\sigma_1 w}{\sigma_1} = \frac{\frac{\sigma_2}{\rho} + \sigma_3 w}{\sigma_3 + \frac{\sigma_1 kr}{\rho}} = \frac{\sigma_2 w + \sigma_3\rho a^2}{\sigma_2} = \lambda$$

Hence

$$(w - \lambda)\,\sigma_1 = 0$$

$$-\frac{kr}{\rho}\lambda\,\sigma_1 + \frac{\sigma_2}{\rho} + (w - \lambda)\,\sigma_3 = 0$$

$$(w - \lambda)\,\sigma_2 + \rho a^2\,\sigma_3 = 0$$

For non-trivial σ_1, σ_2, σ_3

$$\begin{vmatrix} (w - \lambda) & 0 & 0 \\ -\frac{kr}{\rho}\lambda & \frac{1}{\rho} & (w - \lambda) \\ 0 & (w - \lambda) & \rho a^2 \end{vmatrix} = 0$$

Hence

$$\lambda = w \quad \text{(particle path direction)}$$

$$\text{or} \qquad \lambda = w \pm a \quad \text{(ray directions)} \qquad 4.17$$

With $\lambda = w$ take $\sigma_1 = 1$, $\sigma_2 = krw$, $\sigma_3 = 0$

Then equation 4.16 gives

$$\left(\frac{du}{dt}\right)_3 + \frac{kr}{\rho}\left(\frac{dp}{dt}\right)_3 + krw\left(\frac{dw}{dt}\right)_3$$

$$= -u\frac{\partial u}{\partial r} - \frac{1}{\rho}\frac{\partial p}{\partial r} - krwu\frac{\partial w}{\partial r} \qquad 4.18$$

where $$\left(\frac{d}{dt}\right)_3 = \frac{\partial}{\partial\tau} + w\frac{\partial}{\partial z} \qquad 4.19$$

With $\lambda = w \pm a$ take $\sigma_1 = 0,\ \sigma_2 = 1,\ \sigma_3 = \pm 1/(\rho a)$
Equation 4.16 then reduces to

$$\pm\frac{1}{\rho a}\left(\frac{d}{dt}\right)_{1,2}p + \left(\frac{d}{dt}\right)_{1,2}w = \mp a\frac{\partial u}{\partial r} - u\frac{\partial w}{\partial r} \mp \frac{au}{r} \qquad 4.20$$

where $$\left(\frac{d}{dt}\right)_{1,2} = \frac{\partial}{\partial\tau} + (w \pm a)\frac{\partial}{\partial z} \qquad 4.21$$

Equation 4.15 can be written

$$\left(\frac{dS}{dt}\right)_3 = -u\frac{\partial S}{\partial r} \qquad 4.22$$

Equations 4.18, 4.20, 4.22 are in the required near characteristic form. To determine the initial flow pattern shortly after the incident wave first hits the ocean surface we expand the dependent variables in powers of r, for example,

$$p = p_o(t, z) + r\,p_1(t, z)$$

and retain terms up to order r. We then obtain equations for the coefficients written in characteristic form and these are solved in series of the type

$$p_o = p_{oo}\left(\frac{z}{t}\right) + \sqrt{z^2 + t^2}\;p_{o1}\left(\frac{z}{t}\right)$$

starting with the zero order coefficients. These expansions are very similar to those carried out for the spherical explosion by Holt (ref. 8).

The detailed calculations for both these problems are now being carried out and results will be published in later reports.

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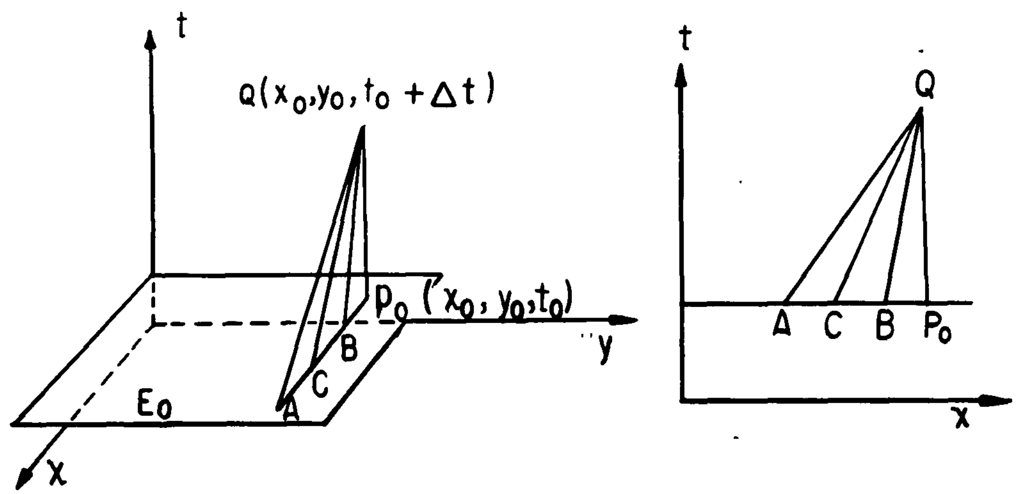

Figure 1. Difference Scheme in Near Characteristics Method

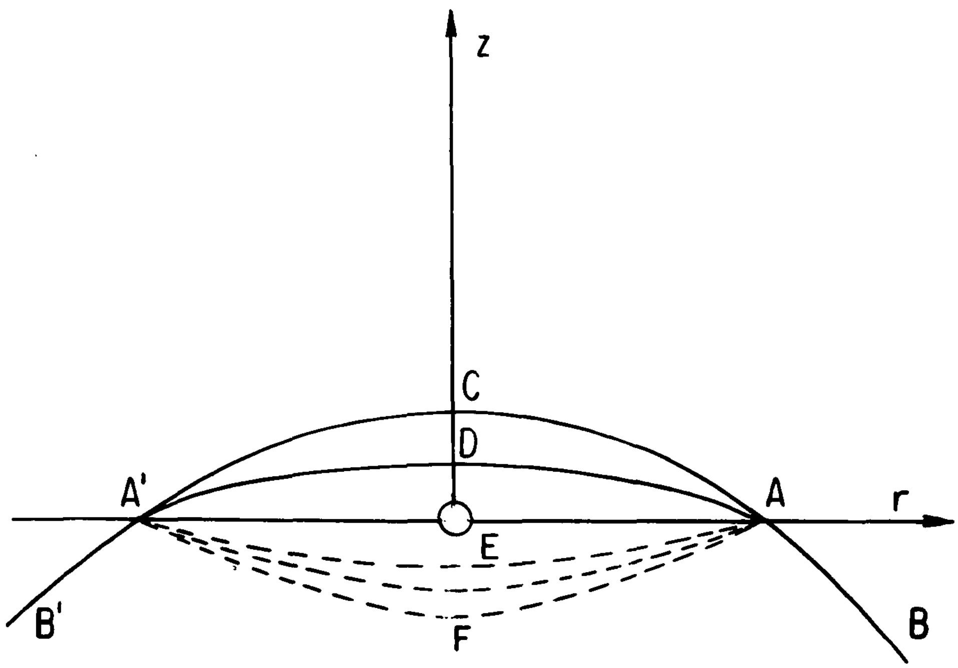

Figure 2. Reflection and refraction of a spherical shock wave.

ACA' Transmitted shock.

ADA' Disturbed ocean surface.

AEA' - AFA' Reflected expansion wave.

AB, A'B' Incident shock.